International Journal of Transportation Engineering and Technology
2023; 9(4): 79-85
http://www.sciencepublishinggroup.com/j/ijtet
doi: 10.11648/j.ijtet.20230904.12
ISSN: 2575-1743 (Print); ISSN: 2575-1751 (Online)

Parameter Estimation and Sensitivity Analysis of Bus Rapid Transit Frequency in Tanzania

Laurencia Ndelamo Massawe[1, *], Oluwole Daniel Makinde[2]

[1]Faculty of Informatics and Technical Education, National Institute of Transport, Dar es Salaam, Tanzania
[2]Faculty of Military Science, Stellenbosch University, Saldanha, South Africa

Email address:
lndelamo@yahoo.com (Laurencia Ndelamo Massawe), makinded@gmail.com (Oluwole Daniel Makinde)
[*]Corresponding author

To cite this article:
Laurencia Ndelamo Massawe, Oluwole Daniel Makinde. Parameter Estimation and Sensitivity Analysis of Bus Rapid Transit Frequency in Tanzania. *International Journal of Transportation Engineering and Technology*. Vol. 9, No. 4, 2023, pp. 79-85.
doi: 10.11648/j.ijtet.20230904.12

Received: September 14, 2023; **Accepted**: October 9, 2023; **Published**: October 28, 2023

Abstract: Mathematical model on a single line was presented based on four equations; operation cost, passenger cost, total cost and Bus Rapid Transit (BRT) service frequency. The analysis of the model shows that the increase of BRT services frequency tends to increase total passenger demand which lead to decrease operational cost and passenger cost in terms of waiting time. In numerical simulation, it is observed that the increase of passenger demand (*p*), tends to increase the frequency of BRT. But absence of Passenger demand (*p*) reduce the BRT frequency which leads to increase operational cost; for example, paying salary for BRT staff, bus services and other cost like office expenses. Furthermore, passenger demand depends on other parameters like the decrease of value of initial bus cost B_{C0}, waiting time W_t, average getting on and ending time per passenger G_{oe}, proportion between average waiting period and the service a_1, hurrying and slowing down at stops and at the junction plus passenger getting on and descending from the bus h_s, increase the value of in-bus time serving B_{ts}, and proportion of average trip length to the total rout length a_2. The specific cases of BRT operation and passenger behaviour can be analysed for their effect on the value of a_1. On the other hand, if movements are large and a timetable of bus plan is published, then passengers change their behaviour and arrive at bus stops a few minutes before the planned bus arrival. This indicates that there is much work to be done for BRT management. BRT management requires organisational efforts, deliberate planning, fund from the public, and coordination between of passengers and staff members.

Keywords: Passenger Demand, Operation Cost, Passenger Cost, BRT Frequency, BRT

1. Introduction

African cities are the fastest-growing in the world, with the region's urban population increasing each year. The accelerated growth of the urban population and the expansion of cities, the increase of economic exchanges as well as environmental concerns have made road traffic [1]. Urbanization can often boost economic growth and open new opportunities; this steady movement of people is also putting great pressure on cities and their transport systems. In the face of these challenges, a growing number of African cities are turning to an innovative form of urban transport known as Bus Rapid Transit (BRT), Bus Rapid Transit (BRT) system can be a solution for high traffic volumes and congestion in large cities as in [2]. In this paper, Bus Rapid Transit is defined as the high-quality bus-based transit system that delivers fast, comfortable, and cost-effective services at metro-level capacities. There are reduced passenger delays due to purchasing fares and boarding processes and faster that reduce traffic congestion caused due to the mixing of buses with other vehicles. The system has brought efficiency and cost-effectiveness to the simplicity of a bus system [3-6]. Most BRT buses operate on dedicated lanes, allowing them to zoom past traffic and offer fast, predictable journey times. Buses only stop at designated stations where passengers typically prepay the fare before boarding, which helps streamline and speed up operations [6]. Services run frequently and are operated by large vehicles and can carry large amounts of people quickly and efficiently.

In African cities, bus rapid transit is a relatively new phenomenon. The first system was opened in 2008 in Lagos, Nigeria and since then, many more BRT systems have been undertaken and are in different stages of development [7-9]. Currently BRT systems are in operation in the following African cities: Lagos (Nigeria) which was opened in March 2008, Johannesburg (South Africa) which was opened in August 2009, Cape Town (South Africa) which was opened in May 2011, Pretoria (South Africa) which was opened in December 2014, George (South Africa) which was opened in August 2015 and Dar es Salaam (Tanzania) which was opened in May 2016 [10] as shown in figure 1.

Figure 1. *BRT in Dar es Salaam (Tanzania).*

Thereafter, followed by Marrakech (Morocco), which was opened in November 2016, Accra (Ghana), which was opened in November 2016, Rustenburg (South Africa) which was opened in September 2022, Casablanca (Morocco), which was opened in 2023 and lastly Agadir (Morocco), which will be opened in 2024. Often BRT is advocated as a cheaper way to build rapid mass transit for African larger cities compared to rail. Implementing these systems is also sometimes conceptualized as a way to initiate wider reform of local bus systems often consisting of privately operated and flexibly run minibuses, sometimes called paratransit or informal transport. These systems are often called "paratransit" because of their flexible schedules, stops and routes, low levels of regulation over competition and formal business practices [11, 12].

The aim of BRT transport economics is to make this coordination effective, ensuring optimal resource allocation to disclose all societal benefits of mass mobility as in [13]. BRT, now seems to be not comfortable to passenger in Tanzania in term of waiting time of the bus. BRT management believe that increasing frequency of a planned transport service, is expensive in terms of operational costs. Nevertheless, since people's time is valuable, reducing the frequency of such a service may also be costly in terms of waiting times for passengers. Outcome of the social welfare-optimal BRT frequency is an important task that seems to have received too little attention in practice. This study addresses this question and exploring what approaches are more suitable for the case of BRT service. This study seeks the simplest model that can adequately describe the given optimization problem. Mathematical model of the square-root rule is proposed, and its accuracy is examined and compared with that of previous version of [14].

2. Model Formulation and Analysis

Optimal Model on a Single Line

In formulating the model, the following assumptions are considered

i) Access time costs is not included as the distance between stops is not an optimization variable in this model.

ii) Bus stop location is fixed.

The total cost of BRT service is comprised of operation and passenger cost as follows:

$$C_{tot} = c\beta + At_a + W_t t_w + B_{ts} t_v \tag{1}$$

Where: c is the cost per BRT unit [Tshs/veh-h], β is the number of buses [veh], t_a, t_w and t_v are total access, waiting and in-bus times of users and A_t, W_t and B_{ts} are the values of access, waiting and in-bus time savings respectively. Bus cost c is modelled as a linear function of bus capacity K [14, 15] and given as

$$c = B_{C_0} + B_{C_1} K \tag{2}$$

Fleet size β is the total cycle time t_c, times the BRT service frequency f [vex/h]. Cycle time is composed of running time that is hurrying and slowing down at stops and at the junction plus passenger getting on and descending from bus, h_s. If P is a total demand [trips/sec.] and G_{oe} is the average time for getting on and ending time per passenger, then cycle time is:

$$t_c = h_s + G_{oe} \frac{P}{f} \tag{3}$$

where P/f is the average number of passengers per bus. Therefore, the operation cost is:

$$C_{op} = \left(B_{C_0} + B_{C_1} K\right)\left(h_s + G_{oe} \frac{P}{f}\right) f \tag{4}$$

If a_1 is the proportion between the average waiting period and the service improvement and a_2 is the proportion of average trip length to the total route length, then passenger cost is:

$$C_P = \left(W_t a_1 \frac{P}{f} + B_{ts} a_2\right)\left(h_s + G_{oe} \frac{P}{f}\right) P \tag{5}$$

In this case, some particular cases of BRT operation and passenger behaviour can be analysed for their impact on the value of a_1. On the other hand, if the frequency of the bus is increased and a time table of bus plan is published, then passengers adjust their behaviour and arrive at bus stops a few minutes before the planed bus arrival. Finally, if θ

Proportion between the climax passenger load of the route and the total passenger demand along the route, we can impose that bus capacity is directly obtained from service frequency as follows:

$$K = \varphi\theta\frac{P}{f} \tag{6}$$

The parameter φ is introduced in equation (6) in order to have reserve capacity that can carry passengers in the middle stations [15]. Therefore, the total cost (C_T) can be written as follows:

$$\begin{aligned} C_T &= C_{op} + C_P \\ C_T &= \left(B_{C_0} + B_{C_1}K\right)\left(h_s + G_{oe}\frac{P}{f}\right)f + \left(W_t a_1\frac{P}{f} + B_{ts}a_2\right)\left(h_s + G_{oe}\frac{P}{f}\right)P \\ \therefore C_T &= \left(B_{C_0} + B_{C_1}\varphi\theta\frac{P}{f}\right)\left(h_s + G_{oe}\frac{P}{f}\right)f + \left(W_t a_1\frac{P}{f} + B_{ts}a_2\right)\left(h_s + G_{oe}\frac{P}{f}\right)P \end{aligned} \tag{7}$$

By differentiating (7) with respect to frequency f, the square root formula for frequency is obtained:

$$C_T = \left(B_{C_0} + B_{C_1}\varphi\theta\frac{P}{f}\right)\left(h_s + G_{oe}\frac{P}{f}\right)f + \left(w_t a_1\frac{P}{f} + B_{ts}a_2\right)\left(h_s + G_{oe}\frac{P}{f}\right)P$$

$$\frac{d}{df}\left(C_T = \left(B_{C_0} + B_{C_1}\varphi\theta\frac{P}{f}\right)\left(h_s + G_{oe}\frac{P}{f}\right)f + \left(W_t a_1\frac{P}{f} + B_{ts}a_2\right)\left(h_s + G_{oe}\frac{P}{f}\right)P\right) = 0$$

$$\begin{aligned} Then\ f^* = {} & \frac{P^2\left(h_s a_1 W_t + \left(\theta\varphi B_{C_1} + B_{ts}a_2\right)G_{oe}\right)}{3^{\frac{1}{3}}\left(9P^3 h_s^2 a_1 B_{C_0}^2 W_t G_{oe} + \sqrt{3}\sqrt{P^6 h_s^3 B_{C_0}^3\left(27h_s a_1^2 B_{C_0} W_t^2 G_{oe}^2 - \left(h_s a_1 W_t + \left(\varphi\theta B_{C_1} + B_{ts}a_2\right)G_{oe}\right)^3\right)}\right)^{\frac{1}{3}}} + \\ & \frac{\left(9P^3 h_s^2 a_1 B_{C_0}^2 W_t G_{oe} + \sqrt{3}\sqrt{P^6 h_s^3 B_{C_0}^3\left(27h_s a_1^2 B_{C_0} W_t^2 G_{oe}^2 - \left(h_s a_1 W_t + \left(\varphi\theta B_{C_1} + B_{ts}a_2\right)G_{oe}\right)^3\right)}\right)^{\frac{1}{3}}}{3^{\frac{2}{3}} B_{C_0}} \end{aligned} \tag{8}$$

$$Thus\ f^* = \frac{P^2\left(h_s a_1 W_t + \left(\theta\varphi B_{C_1} + B_{ts}a_2\right)G_{oe}\right)}{3^{\frac{1}{3}}\beta} + \frac{\beta}{3^{\frac{2}{3}} B_{C_0}}$$

$$where\quad \beta = \left(9P^3 h_s^2 a_1 B_{C_0}^2 W_t G_{oe} + \sqrt{3}\sqrt{P^6 h_s^3 B_{C_0}^3\left(27h_s a_1^2 B_{C_0} W_t^2 G_{oe}^2 - \left(h_s a_1 W_t + \left(\varphi\theta B_{C_1} + B_{ts}a_2\right)G_{oe}\right)^3\right)}\right)^{\frac{1}{3}}$$

BRT service frequency increases with passenger demand, this means that there is decrease of operating cost, and technical parameters and resources values have important implications in the optimal results. For example, if value of initial bus cost B_{C0}, value of waiting time W_t, value of average getting on and ending time per passenger G_{oe}, proportion between average waiting period and the service a_1, value of hurrying and slowing down at stops and at the junction plus passenger getting on and descending from the bus h_s decreased, then BRT frequency increases.

3. Sensitivity Analysis

Here, we illustrate the analytical results of the study by carrying out sensitivity analysis of the model system (4, 5, 7 and 8). Parameter values are estimated to vary within realistic means as shown in table 1.

Table 1. *Parameter description and estimated values.*

Parameters	Parameter description	Parameter estimation
a_1	Proportion between average waiting period and the service	0.17per hour
a_2	Proportion of average trip length to the total route length	0.22 km
B_{C0} & B_{C1}	Bus cost	0.16$ &0.2 $
W_t & B_{ts}	Value of waiting and in-bus time serving	0.11 &0.13per hour

Parameters	Parameter description	Parameter estimation
G_{oe}	Average getting on and ending time per passenger	0.19 per hour
h_s	Hurrying and slowing down at stops and at the junction plus passenger getting on and descending from the bus	0.21per hour
p	Passenger demand	0.23 trip/sec.
φ	reserve capacity that can carry passengers in the middle stations	0.12per vehicle
θ	Proportion between the climax passenger load of the route and the total passenger demand along the route	0.15 trip/h
f	BRT frequency	0.25veh/h

Sensitivity Analysis of the Model Parameters

In this section, different factors for BRT frequency are determined as they help to reduce operation cost and passenger cost.

In order to determine how best, the BRT frequency increased, sensitivity indices of the BRT frequency are calculated to each parameter. These indices tell us the parameters which have high impact on f^* and should be targeted for changing behaviour of BRT operation.

Definition 1: The normalised forward sensitivity index of a variable f that depends differentiable on a parameter g is defined as

$$X_g^f = \frac{\partial f}{\partial g} \times \frac{g}{f} \tag{9}$$

as in [16-19]. Having an explicit formula for f^* in equation (8), we derive an analytical expression for the sensitivity of f^* as $X_g^{f^*} = \frac{\partial f^*}{\partial g} \times \frac{g}{f^*}$ where;

$$f^* = \frac{P^2\left(h_s a_1 W_t + \left(\theta\varphi B_{C_1} + B_{ts} a_2\right) G_{oe}\right)}{3^{\frac{1}{3}}\left(9P^3 h_s^2 a_1 B_{C_0}^2 W_t G_{oe} + \sqrt{3}\sqrt{P^6 h_s^3 B_{C_0}^3\left(27 h_s a_1^2 B_{C_0} W_t^2 G_{oe}^2 - \left(h_s a_1 W_t + \left(\varphi\theta B_{C_1} + B_{ts} a_2\right) G_{oe}\right)^3\right)}\right)^{\frac{1}{3}}} + \frac{\left(9P^3 h_s^2 a_1 B_{C_0}^2 W_t G_{oe} + \sqrt{3}\sqrt{P^6 h_s^3 B_{C_0}^3\left(27 h_s a_1^2 B_{C_0} W_t^2 G_{oe}^2 - \left(h_s a_1 W_t + \left(\varphi\theta B_{C_1} + B_{ts} a_2\right) G_{oe}\right)^3\right)}\right)^{\frac{1}{3}}}{3^{\frac{2}{3}} B_{C_0}}$$

Then analytical expression for the sensitivity of f^* with respect to each parameter can be calculated using a set of reasonable parameter values. The sensitivity indices of f^* with respect to p and B_{C_1} are given by $X_p^{f^*} = \frac{\partial f^*}{\partial p} \times \frac{p}{f^*}$ then $\frac{\partial f^*}{\partial p} \times \frac{p}{f^*} = 0.759805$ and $X_{B_{C_1}}^{f^*} = \frac{\partial f^*}{\partial B_{C_1}} \times \frac{B_{C_1}}{f^*} = 0.0124132$ respectively. Other indices $X_{a_2}^{f^*}, X_{B_{C_0}}^{f^*}, X_{\varphi}^{f^*}, X_{G_{oe}}^{f^*}, X_{B_{ts}}^{f^*}, X_{W_t}^{f^*}, X_{a_1}^{f^*}, X_{h_s}^{f^*}$ *and* $X_{\theta}^{f^*}$ are obtained following the same procedure and highlighted in table 2 as follows:

Table 2. *Sensitivity Indices of Model Parameters to* f^*.

S/N	Parameter Symbol	Sensitivity Index
1	p	0.759805
2	B_{C0}	-0.44003
3	W_t	0.438172
4	G_{oe}	0.295534
5	a_1	0.283523
6	h_s	-0.267388
7	B_{ts}	0.151717
8	a_2	0.0896512
9	φ	0.0206887
10	θ	0.016551
11	B_{C_1}	0.0124132

Interpretation of Sensitivity Indices

By analysing sensitivity indices of the model parameters to f^*, it is observed that the most sensitive parameter is p, this indicate that the passenger demand (p) has high impact of increasing the frequency of BRT. Passenger demand (p) depends on other parameters like value of initial bus cost B_{C0}, waiting time W_t, average getting on and ending time per passenger G_{oe}, proportion between average waiting period and the service a_1, hurrying and slowing down at stops and at the junction plus passenger getting on and descending from the bus h_s, Value of in-bus time serving B_{ts}, proportion of average trip length to the total route length a_2, reserve capacity that can carry passengers in the middle stations φ, Proportion between the climax passenger load of the route and the total passenger demand along the route θ, and bus cost B_{C1} have the least effect on the increase of frequency of BRT and passenger demand as well.

Numerical Simulations

Here analytical results are illustrated by carrying out numerical simulations of the model equations 4, 5, 7 and 8. Parameter values are estimated to vary within realistic means as shown in (10).

B =0.16, B_{C1} = 0.2, φ =0.6, θ=0.15, f=0.25, h_s=0.21, G_{oe}=0.19, a_1=0.17 a_2 =0.22, B_{ts} =0.13, W_t =0.11, P=0.23. (10)

The variation of BRT frequency in different values B_{C0}, W_t, G_{oe}, a_1 and h_s are shown in Figures 2-6. It is observed that as B_{C0} increases, the BRT frequency decrease, this is due to the decrease of passengers' demand because of high cost.

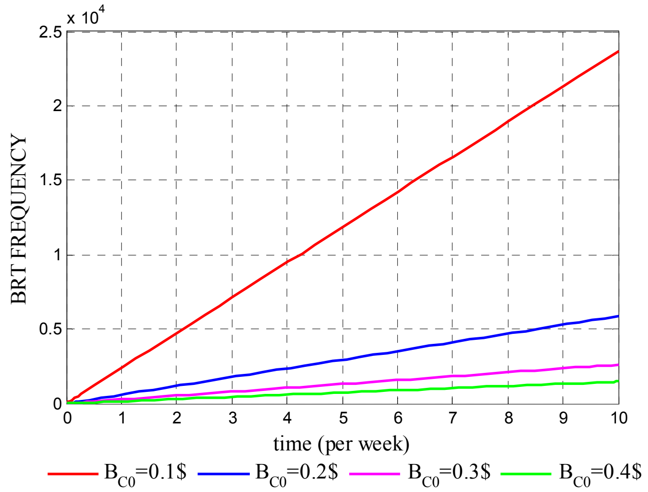

Figure 2. *BRT frequency against time with increasing B_{C0}.*

Figure 3, illustrates that the increase of W_t led to decrease of BRT frequency but when W_t is reduced the passenger demand became high leading to the increase of BRT frequency and consequently reduced the operational cost and passenger cost in terms of waiting time.

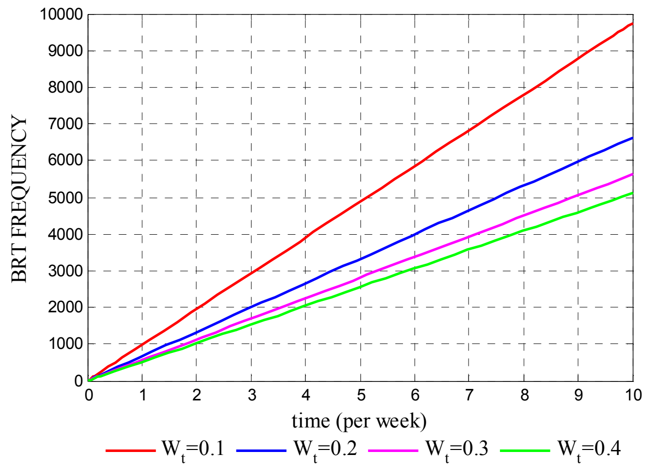

Figure 3. *BRT frequency against time with increasing w_t.*

Figure 4 presents the influence of increasing the average getting on and ending time per passenger G_{oe} , it was observed that as G_{oe} increases BRT frequency decreases. Then, as proportion between average waiting period and the service a_1, increases, the BRT frequency decreases as shown in figure 5. In this case, some particular cases of BRT operation and passenger behaviour can be analysed for their impact on the value of a_1 by publish the planned bus time table which led passengers to alter their behaviour and arrive at bus stops a few minutes before the planned bus arrival.

The effect of hurrying and slowing down and at the junction plus passenger getting on and descending from the bus, h_s, is presented in figure 6. It was observed that when h_s is increased, then the BRT frequency is decreased due to wastage of time on the way to destination.

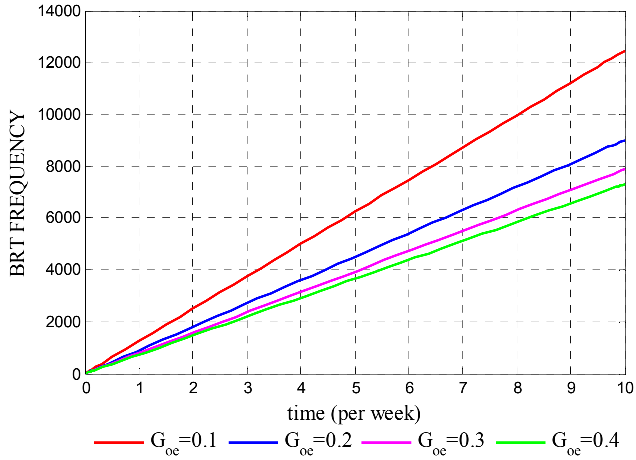

Figure 4. *BRT frequency against time with increasing G_{oe}.*

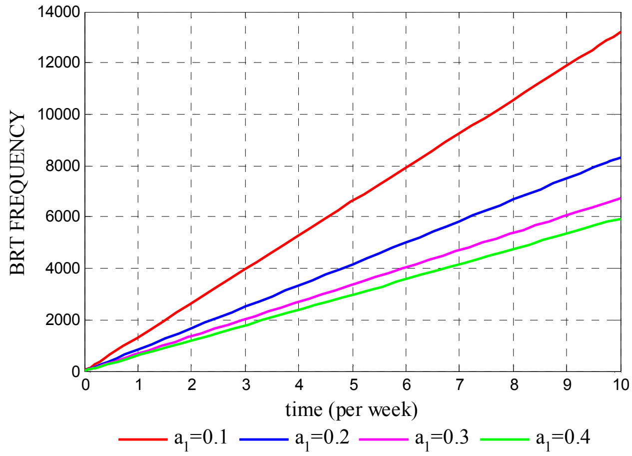

Figure 5. *BRT frequency against time with increasing a_1.*

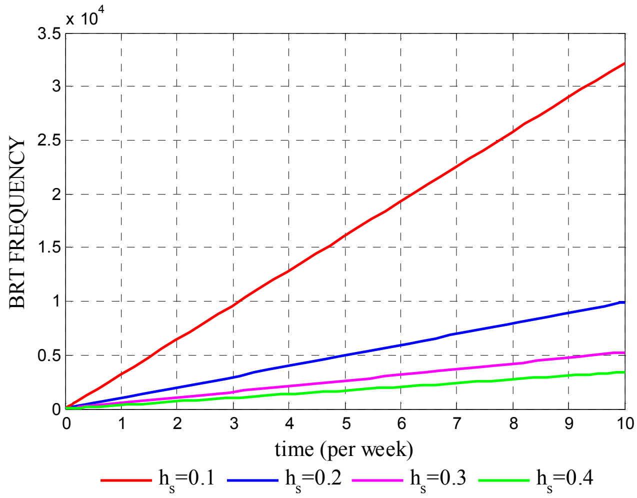

Figure 6. *BRT frequency against time with increasing h_s.*

The effect of different values of B_{ts} and a_2 are shown in Figures 7-8. It is observed that as B_{ts} increases, the BRT frequency increases as a result, attract passengers and consequently leads to the increase of BRT frequency. In figure 8 it has been observed that when value of proportion of average trip length to the total route length a_2 is increased then the BRT frequency is also increased.

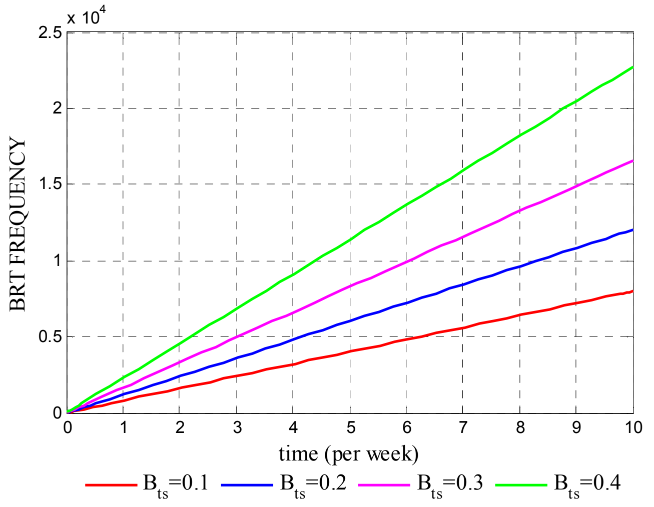

Figure 7. *BRT frequency against time with increasing B_{ts}.*

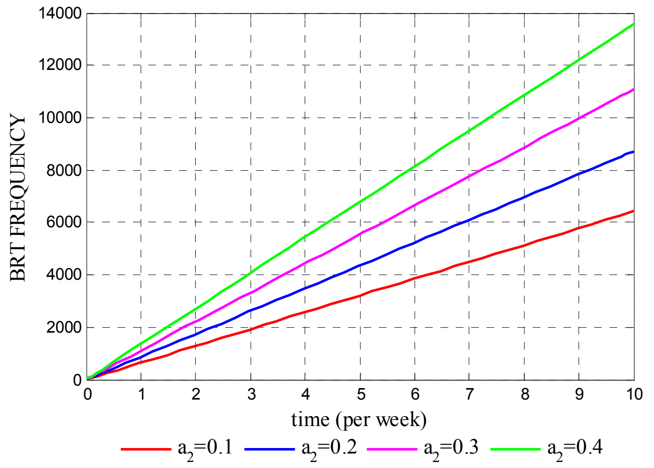

Figure 8. *BRT frequency against time with increasing a_2.*

4. Discussion and Conclusion

4.1. Discussion

Mathematical model has been analysed to study the effect of BRT service frequency in Transport services. The analysis of the model shows that the increase of BRT services frequency tends to increase a total demand of passengers which leads to decrease operating cost and passengers' cost in term of waiting time. Sensitivity analysis was conducted to obtain parameters with high impact on increasing BRT frequency and should be targeted for changing BRT operational behaviour. It has been observed that the increase of passenger demand (p), leads to increase the frequency of BRT. But absence of Passenger demand (p) reduces the BRT frequency which leads to the increase of operational cost such as paying salary for BRT staff, BRT services and other cost like office expenses. Furthermore, numerical simulations were used to demonstrate the impact of parameters on the BRT frequency. The results show that the decrease of value of initial bus cost B_{C0}, waiting time W_t, average getting on and ending time per passenger G_{oe}, proportion between average waiting period and the service a_1, hurrying and slowing down at stops and at the junction plus passenger getting on and descending from the bus h_s lead to the increase of BRT frequency. Additionally, the increase of the value of in-bus time serving B_{ts}, and proportion of average trip length to the total route length a_2 tends to increase BRT frequency.

Therefore, this indicates that there is much work to be done, BRT management requires organisational efforts, deliberate planning, fund from the public, and coordination between passengers and staff members so as to increase the eagerness of people to use BRT.

4.2. Conclusion

Model on a Single Line was presented based on four equations; operation cost, passenger cost, total cost (combination of passenger cost and operation cost) and BRT service frequency. The simulation shows that reducing waiting time of passengers has a positive impact on the increase of BRT frequency.

References

[1] M. Hjaeyj, H. Mosbahi. Assessment of Level of Service and Acoustic Impacts of Road Segment in Sousse City. *International Journal of Transportation Engineering and Technology*. Vol. 8, No. 2, 2022, pp. 24-29. doi: 10.11648/j.ijtet.20220802.11.

[2] M. Rizzo, Taken for a Ride. In Grounding Neoliberalism, Precarious Labour, and Public Transport in an African Metropolis, Oxford University Press: Oxford, UK, ISBN 2017; 978-0-19-879424-0. [Google Scholar].

[3] T. Deng, J. D. Nelson, Recent Developments in Bus Rapid Transit: A Review of the Literature. Transport Reviews 31, 2011, 69–96. https://doi.org/10.1080/0 1441647.2010.492455.

[4] ITDP, Bus Rapid Transit Planning Guide. Institute for Transportation & Development Policy, New York 2007.

[5] H. Suzuki, R. Cervero, K. Iuchi, Transforming Cities with Transit: transit and land-use integration for sustainable urban development. The World Bank, Washington, D. C., 2013.

[6] D. Hidalgo, L. Gutiérrez, BRT and BHLS around the world: Explosive growth, large positive impacts and many issues outstanding. Research in Transportation Economics 39, 2013, 8–13, https://doi.org/10.1016/j. retrec.2012.05.018.

[7] F. Krüger, A. Titz, R. Arndt, F. Groß, F. Mehrbach, V. Pajung, L. Suda, M. Wadenstorfer, L. Wimmer, The Bus Rapid Transit (BRT) in Dar es Salaam: A Pilot Study on Critical Infrastructure, Sustainable Urban Development and Livelihoods, Sustainability 2021, 13 (3), 1058; https://doi.org/10.3390/su13031058.

[8] BRTDATA. ORG, Global BRT Data, Available online: https://brtdata.org/ (accessed on 30 July 2020).

[9] A. Wood, Moving policy: Global and local characters circulating bus rapid transit through South African cities. Urban Geogr. 35, 2014, 1238–1254. [Google Scholar] [CrossRef].

[10] Jacobsen, M. (2021) Co-producing urban transport systems: adapting a global model in Dar es Salaam, Sustainability: Science, Practice and Policy, 17:1, 47-61. https://doi.org/10.1080/15487733.2020.1862545

[11] R. Behrens, D. McCormick, D. Mfinanga, Paratransit in African cities: operations, regulation and reform, Routledge 2016.

[12] UITP, Overview of public transport in Sub-Saharan Africa. International Association of Public Transport, Brussels, 2008.

[13] D. Horcher and A. Tirachini, A review of public transport economics. Economics of Transportation 25, 2021, 100196.

[14] H. Mohring, Optimization and scale economies in urban bus transportation. American Economic Review, 62, 1972, 591-604.

[15] A. Tirachini, D. Hörcher and E. Verhoef, eds., Public Transport: design, scale and pricing. Chapter in Handbook on Transport Pricing and Financing, Chapter June 2022.

[16] P. Driessche and J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Mathematical Biosciences, 180, 2002, 29–48.

[17] L. N. Massawe, E. S. Massawe, O. D. Makinde. Modelling Infectiology of Dengue Epidemic. *Applied and Computational Mathematics*. Vol. 4, No. 3, 2015, pp. 192-206. doi: 10.11648/j.acm.20150403.22.

[18] A. S. Eegunjobi, O. D. Makinde, Impact of Corruption in a Society with Exposed Honest Individuals: A Mathematical Model, Asia Pac. J. Math. 2023, 10-18.

[19] Diaby, M., Sène, M. and Sène, A. (2019) Global Transmission Dynamics of a Schistosomiasis Model and Its Optimal Control. Applied Mathematics, 10, 397-418. https://doi.org/10.4236/am.2019.106029